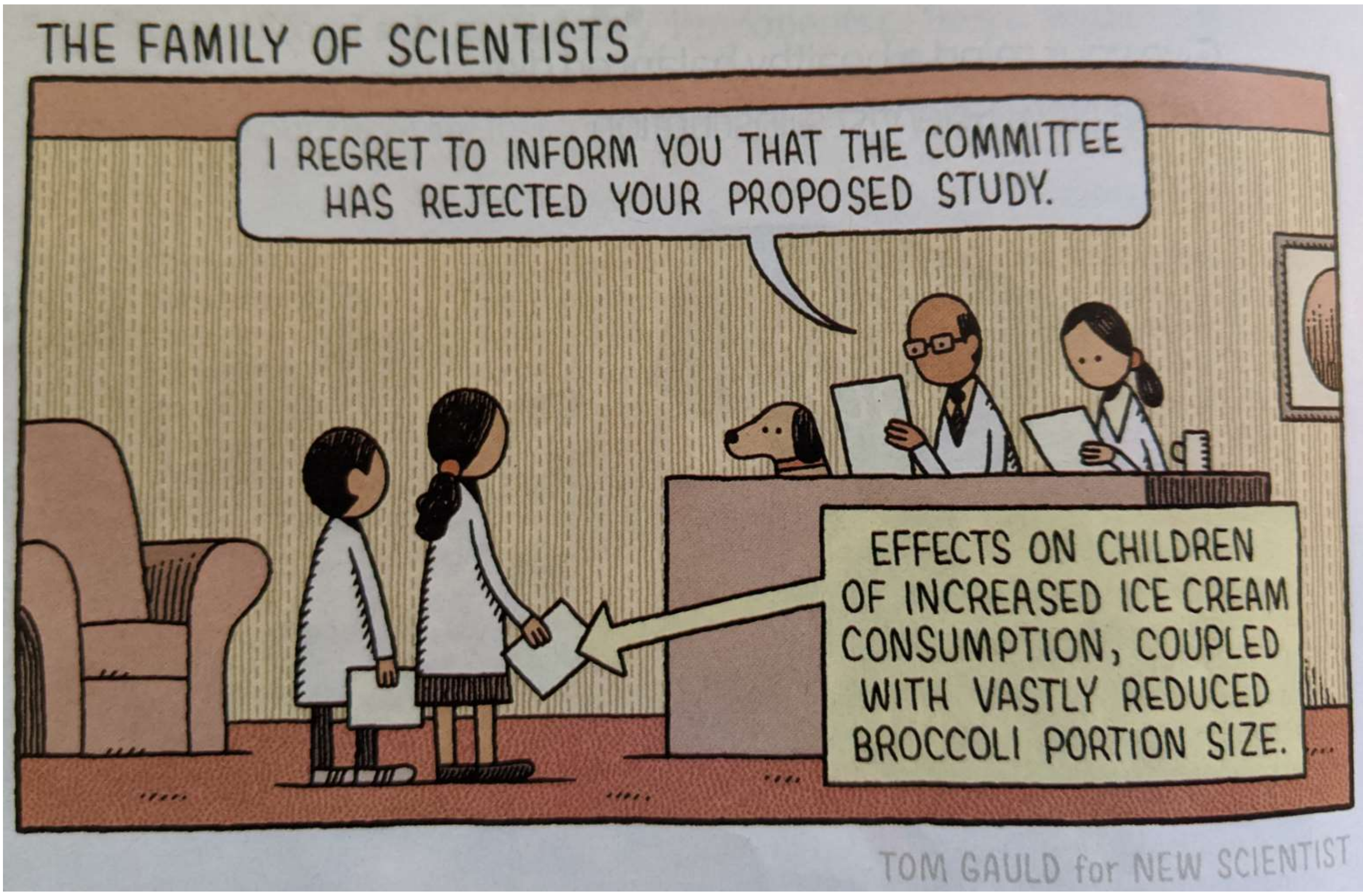
Intro

A few years ago, 120 researchers and 400 undergraduates were presented with six false interpretations of a confidence interval.

On average, the group endorsed more than three of the six falsehoods, with researchers worse than undergrads.

If PhD researchers struggle with inference, what hope is there for senior students studying Applications to identify an association or for those studying Methods and Specialist to interpret a confidence interval?



Australian Senior Curriculum :: Inference Topics

Statistical Inference - drawing conclusions about a population from a sample.

Methods Year 11

Conditional probability and independence

Can we infer from data that events are independent?

Applications Year 12

Identifying and describing associations between two categorical variables

Can we infer from differences across categories that an association exists?

Identifying and describing associations between two numerical variables

Can we infer from the correlation coefficient that a linear association exists between two variables?

Methods Year 12

Random sampling

Why sample? Why might our sample not be random? Does it matter?

Confidence interval for a proportion

Can we infer a population proportion from a sample proportion?

Specialist Year 12

Confidence interval for a mean

Can we infer a population mean from a sample mean?

Methods Unit 1 - Independence of events

Conditional probability and independence:

use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.

Question 15 **(8 marks)**

In a school survey of 197 students in Year 11 and Year 12, it was observed that 75 of the 96 Year 12 students studied a science subject and that 10 students in Year 11 did not study a science subject.

(a) If one student is selected at random from those surveyed, determine the probability that

(ii) they studied a science subject. (1 mark)

(iv) they studied a science subject given that they were in Year 11. (2 marks)

(b) Without calculating any further probabilities, is there any indication that studying a science subject is independent of Year? Justify your answer. (2 marks)

Observed	Year 11	Year 12	Total
Science	91	75	166
Non-science	10	21	31
Total	101	96	197

$$(ii)\ \ P(Sc) = 166 \div 197 \approx 0.84$$

$$(iv)\ P(Sc|Yr11) = 91 \div 101 \approx 0.90$$

How close for independence?

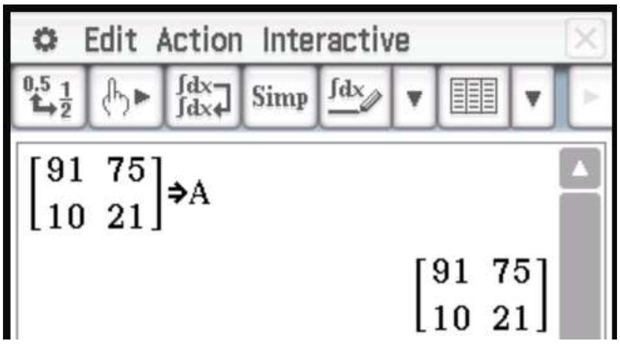
NB If $A * B$ denotes A is independent of B then it follows that $B * A, A * \bar{B}, \bar{A} * B$ and so on.

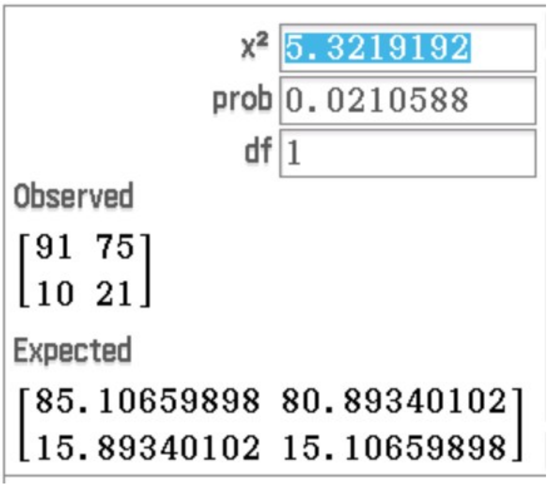
Expected	Year 11	Year 12	Total
Science			166
Non-science			31
Total	101	96	197

$$n(Sc \cap Yr11) = \frac{101 \times 166}{197} \approx 85$$

Recall the **chi-square test of independence** ?

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$





ClassPad: Stats, Calc, Test, χ^2 Test.
Matrix A

If 'survey' a census then we would have to answer NOT independent as $P(Sc) \neq P(Sc|Yr11)$.

But if a sample, then we have a case for independence as our test indicates such a difference would occur by chance about 2% of the time.

Col %	Year 11	Year 12
Science	90%	78%
Non-science	10%	22%
Total	100%	100%

Row %	Year 11	Year 12	Total
Science	55%	45%	100%
Non-science	32%	68%	100%

Applications U3 Categorical Variables

Identifying and describing associations between two categorical variables:

- construct two-way frequency tables and determine the associated row and column sums and percentages
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data.

Example: 2002 General Social Survey about gun ownership and gun laws

Owns Gun	Favors Gun Law	Opposes Gun Law	All
No	527	72	599
Yes	206	102	308
All	733	174	907

Counts are difficult to interpret, especially with unequal numbers of observations in the rows and columns. Percentages are more useful than counts for describing how two categorical variables are related.

Column percents for gun ownership and feelings about gun laws.

Owns Gun	Favors Gun Law	Opposes Gun Law
No	71.90	41.38
Yes	28.10	58.62
All	100.00	100.00

Conditional Percentages as evidence of an association
Definition **: Two categorical variables are associated in a sample if at least two rows (columns) noticeably differ in the pattern of row (column) percentages.**

For gun ownership, the two columns clearly have different sets of columns percentages. Gun ownership and opinion about gun laws are associated.

Do we need an explanatory and response variable?

Can we have an explanatory and response variable?

TAS GEN 2018 Q1

Question 1 (approximately 6 minutes)

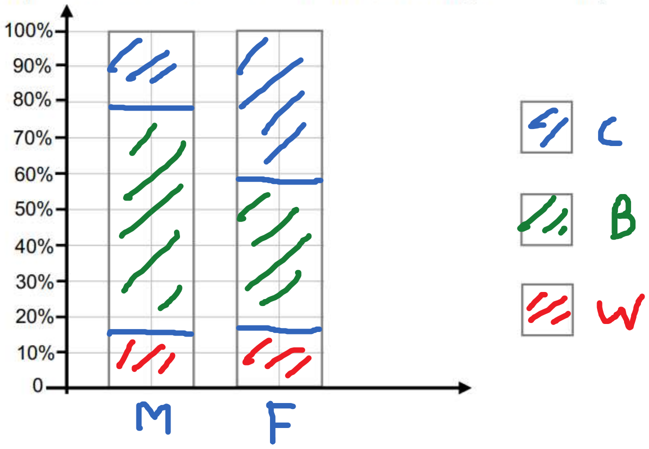
A group of students were surveyed about the type of transport that they used to get to school. A summary table showing the results of this survey is shown below.

	Males	**Females**
Walk	11	18
Bus	42	40
Car	14	40
Totals	67	98

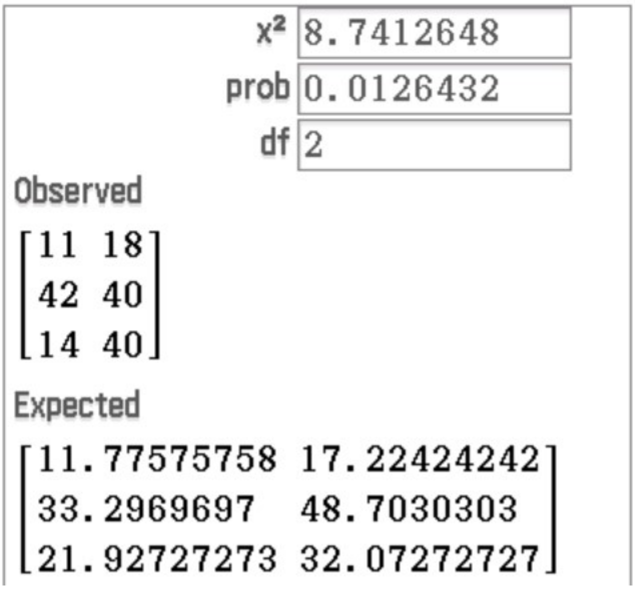
(a) Complete the table below showing the females data in percentage terms. (1 mark)

	Males %	**Females %**
Walk	16	18
Bus	63	41
Car	21	41

(b) Complete the divided bar chart below, showing the data in percentages from part (a). (2 marks)



(c) Using the information given and your answers from parts (a) and (b), comment on any **associations** between the variables presented. (3 marks)



Apps 2017 Q5

Question 5 **(9 marks)**

A group of university students was asked the question 'Does full attendance at school lead to an improved examination result?'

The results are summarised below.

	Agree	Disagree	Undecided
Male under 20 years	8	22	6
Female under 20 years	6	20	8
Male 20 to 25 years	26	7	3
Female 20 to 25 years	30	9	5
Male over 25 years	24	3	2
Female over 25 years	18	2	1

(a) Complete the two-way table below. (2 marks)

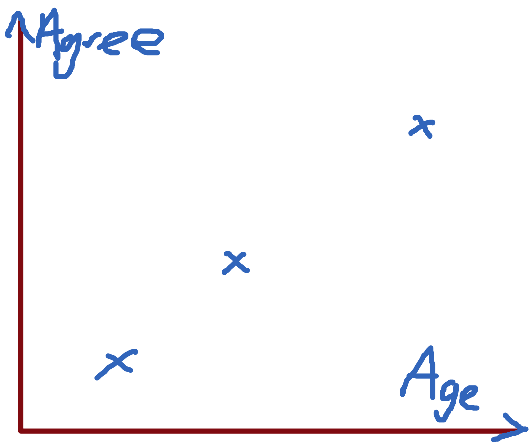
	Agree	Disagree	Undecided
Under 20	14		
20–25			
Over 25			3

(b) State the explanatory variable for these data. (1 mark)

(d) Use the data to determine one association between the variables. Describe the association and explain your reasoning. (2 marks)

	Percentages		
	Agree	**Disagree**	**Undecided**
Under 20	**20**	60	**20**
20–25	**70**	20	**10**
Over 25	84	**10**	**6**

Solution
As age increases the percentage of students who agree increases. Percentages in the **Agree** column are increasing with age. There are other possibilities.
Specific behaviours
✓ correctly states an association ✓ gives reasoning



Categorical data has morphed into numerical data, ready for bivariate analysis!

Assn Numerical Variables - Apps Unit 3

Identifying and describing associations between two numerical variables:

- construct a scatterplot to identify patterns in the data suggesting the presence of an association
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association.

SCATTERPLOTS

Describing a Scatterplot

- Form: pattern type (*i.e.* linear or non-linear).
- Direction: where the points tend towards
  - Positive: from bottom-left to top-right
  - Negative: from top-left to bottom-right
- Strength: how closely the points follow a linear pattern (*e.g.* perfect, strong, etc.).

Value of r	Strength	Direction
$r = 1$	Perfect	Positive
$0.75 \leq r < 1$	Strong	Positive
$0.5 \leq r < 0.75$	Moderate	Positive
$0.25 \leq r < 0.5$	Weak	Positive
$-0.25 \leq r < 0.25$	None	None
$-0.5 \leq r < -0.25$	Weak	Negative
$-0.75 \leq r < -0.5$	Moderate	Negative
$-1 < r < -0.75$	Strong	Negative
$r = -1$	Perfect	Negative

Strong, negative, non-linear assn.

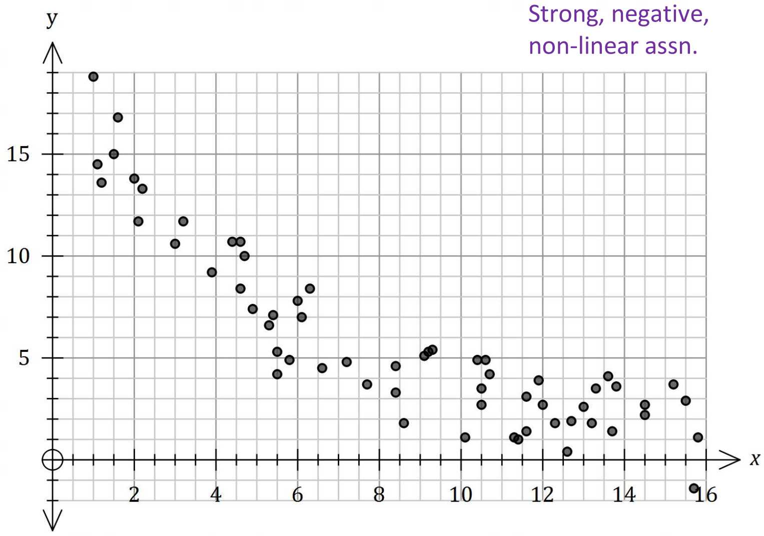


Table 1
Interpretation of the Pearson's and Spearman's correlation coefficients.

Correlation Coefficient		Dancey & Reidy (Psychology)	Quinnipiac University (Politics)	Chan YH (Medicine)
+1	−1	Perfect	Perfect	Perfect
+0.9	−0.9	Strong	Very Strong	Very Strong
+0.8	−0.8	Strong	Very Strong	Very Strong
+0.7	−0.7	Strong	Very Strong	Moderate
+0.6	−0.6	Moderate	Strong	Moderate
+0.5	−0.5	Moderate	Strong	Fair
+0.4	−0.4	Moderate	Strong	Fair
+0.3	−0.3	Weak	Moderate	Fair
+0.2	−0.2	Weak	Weak	Poor
+0.1	−0.1	Weak	Negligible	Poor
0	0	Zero	None	None

So many handy guides to 'strength' of correlation.

But what about sample size?

$n = 2$ and $r = \pm 1$?

Valid?!

Google

strength of correlation coefficients pearson

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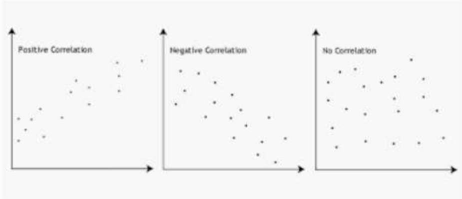
interpretation | spss | moderate | weak | table | grip strength | values | pcc | spearman | scatter plot | lin

Intention to continue using WeChat

Social Inclusion	Pearson's correlation	0.479**
	Sig. (2-tailed)	0.000
Sex	Pearson's correlation	0.097
	Sig. (2-tailed)	0.339
Friendship	Pearson's correlation	0.443**
	Sig. (2-tailed)	0.000
Entertainment	Pearson's correlation	0.443**
	Sig. (2-tailed)	0.000
Romantic Relationships	Pearson's correlation	0.397**
	Sig. (2-tailed)	0.000
People Nearby	Pearson's correlation	0.442**
	Sig. (2-tailed)	0.000

**Correlation is significant at the 0.01 level (2-tailed)

Pearson's correlation coefficient ...
researchgate.net



Pearson Product-Moment Correlation ...
statistics.laerd.com

Range	Strength of association
0	No association
0 to ±0.25	Negligible association
±0.25 to ±0.50	Weak association
±0.50 to ±0.75	Moderate association
±0.75 to ±1	Very strong association
±1	Perfect association

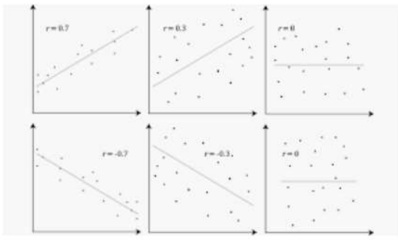
Pearson Correlation Coefficient (pcc ...
chegg.com

Number of performance monitoring measures (X)	Average no. of accidents reported per 1 million kilometres (Y)	X - Mx	Y - My	(X - Mx)2	(Y - My)2	(X - Mx)(Y - My)
1	3.5	2.500	0.817	6.25	0.667	-2.042
2	4.9	-1.5	2.217	2.25	4.914	-3.325
3	1.8	-0.5	-0.883	0.25	0.780	0.442
4	1.3	0.5	-1.383	0.25	1.914	-0.692
5	2.6	1.5	-0.083	2.25	0.007	-0.125
6	2	2.5	-0.683	6.25	0.467	-1.708
		Mx 3.50	My 2.68	Sum 17.50	Sum 8.74	Sum -7.45

Pearson Correlation Coefficient ...
researchgate.net

	Coefficient, r	
Strength of Association	Positive	Negative
Small	.1 to .3	-0.1 to -0.3
Medium	.3 to .5	-0.3 to -0.5
Large	.5 to 1.0	-0.5 to -1.0

What are real strengths of association ...
stats.stackexchange.com



Pearson Product-Moment Correlation ...
statistics.laerd.com

Evaluating Strength. Numerically.

- Correlation coefficients are between -1 and 1.

Sign of correlation coefficient	Strong	Moderate	Weak	Very weak or None
+ values	0.5 to 1.0	0.3 to 0.49	0.1 to 0.29	0 to 0.09
- values	-1.0 to -0.5	-0.49 to -0.3	-0.29 to -0.1	-0.09 to 0

- A coefficient of zero means NO relationship.

Correlations using SPSS
slideshare.net

Correlation Coefficient Interpretation

Correlation Analysis

Coefficient Range	Strength of Relationship
0.01 - 0.20	Very weak
0.21 - 0.40	weak
0.41 - 0.60	Moderate
0.61 - 0.80	Strong
0.80 - .99	Very strong

Correlation Analysis. A measure of ...
slideplayer.com

using WeChat

	Pearson's correlation
	Sig. (2-tailed)
	Pearson's correlation
	Sig. (2-tailed)
	Pearson's correlation
	Sig. (2-tailed)
	Pearson's correlation
	Sig. (2-tailed)
ips	Pearson's correlation
	Sig. (2-tailed)
	Pearson's correlation
	Sig. (2-tailed)

ificant at the 0.01 level (2-tailed)

Pearson's correlation coefficient ...
researchgate.net

Correlation Coefficient Interpretation Guideline

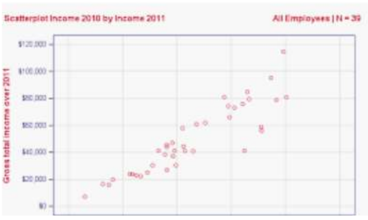
Rule of thumb:

- $0.0 = |r|$: no correlation
- $0.0 < |r| < 0.2$: very weak correlation
- $0.2 \leq |r| < 0.4$: weak correlation
- $0.4 \leq |r| < 0.6$: moderately strong correlation
- $0.6 \leq |r| \leq 0.8$: strong correlation
- $0.8 \leq |r| < 1.0$: very strong correlation
- $1.0 = |r|$: perfect correlation

correlation coefficient
slideshare.net

Table 1 Rule of Thumb for Interpreting the Size of a Correlation Coefficient[a]

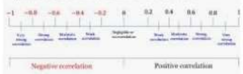
Size of Correlation	Interpretation
.90 to 1.00 (-.90 to -1.00)	Very high positive (negative) correlation
.70 to .90 (-.70 to -.90)	High positive (negative) correlation
.50 to .70 (-.50 to -.70)	Moderate positive (negative) correlation
.30 to .50 (-.30 to -.50)	Low positive (negative) correlation
.00 to .30 (.00 to -.30)	negligible correlation



	Jamar	Myogrip	Vigorimeter
Hand Circumference	0.808*	0.821	0.693
	0.806**	0.824	0.755
Forearm	0.757	0.750	0.604
Circumference	0.824	0.808	0.703
Wrist Circumference	0.718	0.730	0.591
	0.753	0.791	0.639
Weight	0.718	0.707	0.540
	0.717	0.744	0.574
Height	0.684	0.715	0.691
	0.673	0.708	0.680
Hand length	0.672	0.692	0.646
	0.632	0.739	0.644
Palm length	0.621	0.657	0.607
	0.608	0.686	0.595
Age	-0.061	-0.047	-0.136
	(p=0.309)	(p=0.350)	p=0.132)
	0.029	-0.005	-0.023
	(p=0.406)	(p=0.484)	(p=0.424)

Correlation Coefficient Interpretation Guideline

The correlation coefficient (r) ranges from -1 (a perfect negative correlation) to 1 (a perfect positive correlation). In short, $-1 \leq r \leq 1$.



Excel simulation

	A	B
1	63	92
2	97	48
3	66	88
4	16	94
5	36	79
6	66	98
7	90	41
8	29	91

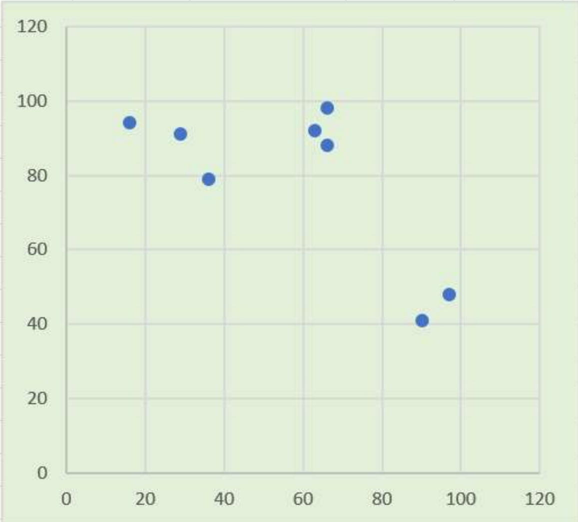
Size	# Samples
8	20

r (strongest out of all samples)

-0.704

Refresh

Refresh will randomly fill columns A and B with <Size> pairs of values and repeat <# Samples> times, recording the strongest value of r out of all the samples and displaying the corresponding data set.

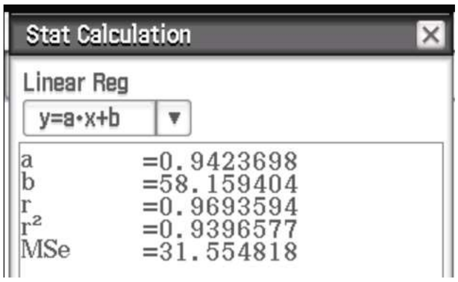


Apps 2017 Q9 (parts of)

Question 9 **(13 marks)**

The World Health Organisation produces tables showing Child Growth Standards. The median lengths (cm) for girls at various times during the first five years of life are shown below.

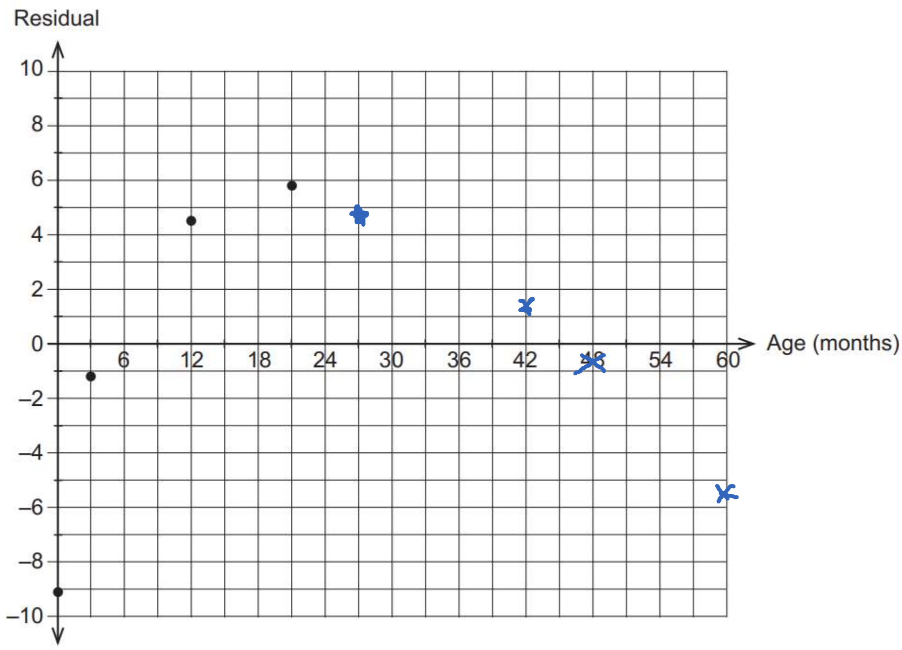
Age (months)	0	3	12	21	27	42	48	60
Median length (cm)	49.1	59.8	74.0	83.7	88.3	99.0	102.7	109.4
Predicted length (cm)	58.2	61.0	69.5	77.9	A	97.7	B	114.7
Residual	–9.1	–1.2	4.5	5.8	4.7	1.3	C	D

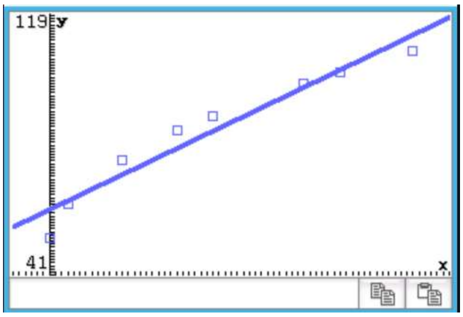


(a) (iv) Given that the correlation coefficient is 0.97, describe the association between age and median length in terms of its direction and strength. (2 marks)

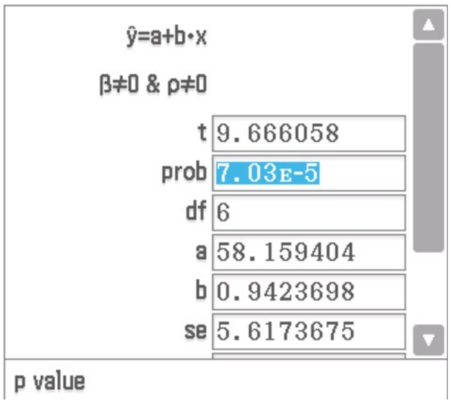
Solution
Positive and strong
Specific behaviours
✓ states direction of association
✓ states strength of association

(b) (ii) Hence, complete the scattergraph of the residuals against age on the axes below by plotting the last four residual values. (2 marks)





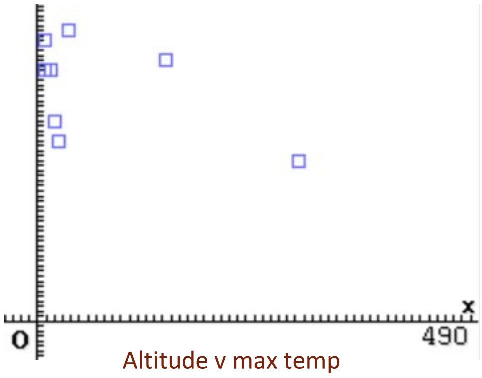
(iii) Use the residual plot to assess the appropriateness of fitting a linear model to the data. (2 marks)



Question 16 **(7 marks)**

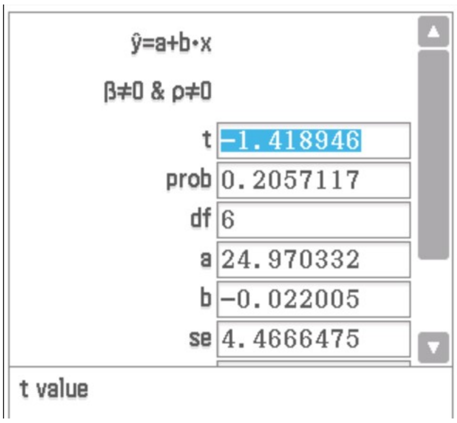
The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (T)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25



Altitude v max temp

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

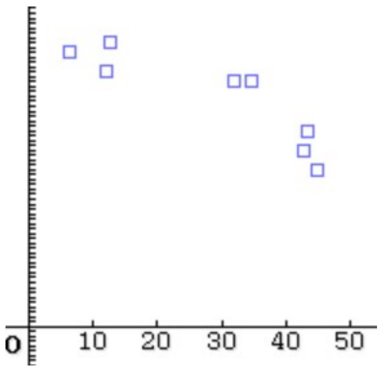


(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)



Professor Bumbledorf reports that

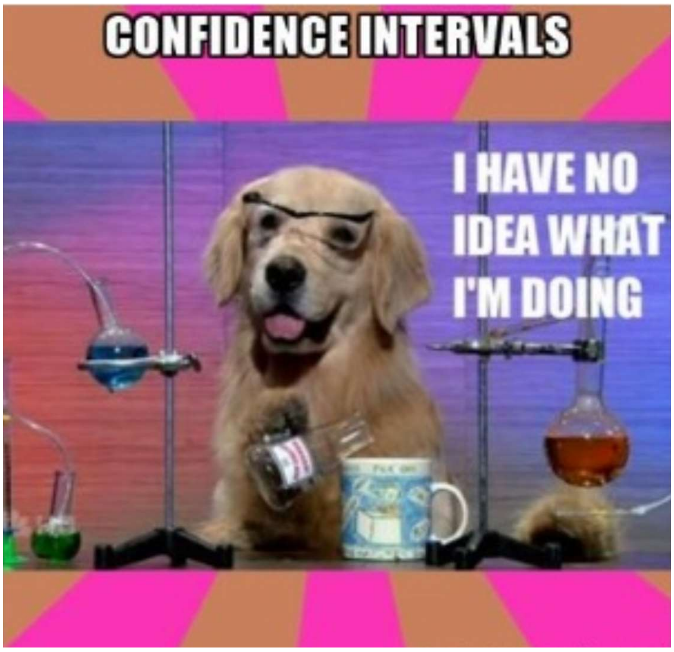
the 95% confidence interval for the mean ranges from 0.1 to 0.4

Please mark each of the statements below as "true" or "false". False means that the statement does not follow logically from Bumbledorf's result. Also note that all, several, or none of the statements may be correct:

1. The probability that the true mean is greater than 0 is at least 95%. ☐True ☐False
2. The probability that the true mean equals 0 is smaller than 5%. ☐True ☐False
3. The "null hypothesis" that the true mean equals 0 is likely to be incorrect. ☐True ☐False
4. There is a 95% probability that the true mean lies between 0.1 and 0.4. ☐True ☐False
5. We can be 95% confident that the true mean lies between 0.1 and 0.4. ☐True ☐False
6. If we were to repeat the experiment over and over, then 95% of the time the true mean falls between 0.1 and 0.4. ☐True ☐False

Table 1 Percentages of students and teachers endorsing an item

Statement	First Years ($n = 442$)	Master Students ($n = 34$)	Researchers ($n = 118$)
The probability that the true mean is greater than 0 is at least 95 %	51 %	32 %	38 %
The probability that the true mean equals 0 is smaller than 5 %	55 %	44 %	47 %
The "null hypothesis" that the true mean equals 0 is likely to be incorrect	73 %	68 %	86 %
There is a 95 % probability that the true mean lies between 0.1 and 0.4	58 %	50 %	59 %
We can be 95 % confident that the true mean lies between 0.1 and 0.4	49 %	50 %	55 %
If we were to repeat the experiment over and over, then 95 % of the time the true mean falls between 0.1 and 0.4	66 %	79 %	58 %



Statements 1-4 assign probabilities to parameters or hypotheses.
Statements 4-6 include specific boundaries.

A correct statement is

"If we were to repeat the experiment over and over, then 95 % of the time the confidence intervals contain the true mean."

Once an interval is computed for a particular sample, it either contains the true mean or it does not; there is no longer anything random about it.

Level of Confidence

What does the level of confidence mean?

- **Take 100 samples from the same population**
- **Calculate sample statistic e.g. proportion**
- **Use the 100 sample proportions to construct 100 CI's**
- **95 of these 100 CI's are expected to contain the true population parameter**

Confidence is in the method - not any particular interval.

Assumptions made when constructing CI's (using z-scores)

Random - data comes from random sample of population

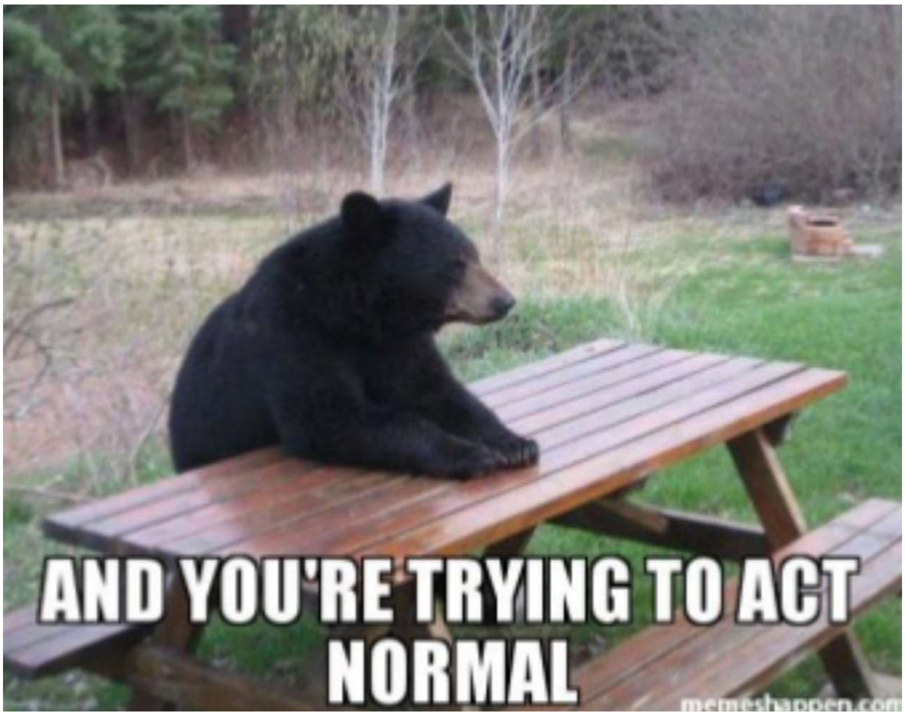
Normal - sampling distribution is normal

Independent - individual observations are independent
(i.e. sample size < 10% of population)

Normal assumption

For proportion, commonly check that $np \geq 10$ and $n(1-p) \geq 10$.
(Note both courses require <u>approximate</u> CI's based on normality)

For mean, if population is normal then OK, otherwise central limit theorem says better approximation as n increases… $n \geq 30$…



Methods Unit 4 - CI for proportion

Topic 3: Interval estimates for proportions

Random sampling:

- understand the concept of a random sample
- discuss sources of bias in samples, and procedures to ensure randomness

Confidence intervals for proportions:

- the concept of an interval estimate for a parameter associated with a random variable
- use the approximate confidence interval $\left(\hat{p} - z\sqrt{(\hat{p}(1-\hat{p})/n}, \ \hat{p} + z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution
- define the approximate margin of error $E = z\sqrt{(\hat{p}(1-\hat{p})/n}$ and understand the trade-off between margin of error and level of confidence
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p.

Methods 2019 Q8

Question 8 **(7 marks)**

Big Foods is a large supermarket company. The manager of Big Foods wants to estimate the proportion of households that do the majority of their grocery shopping in their stores.

A junior staff member at Big Foods conducted a survey of 250 randomly-selected households and found that 56 did the majority of their grocery shopping at a Big Foods store.

(a) (i) Calculate the sample proportion of households who did the majority of their grocery shopping at Big Foods. (1 mark)

$$\hat{p} = 56 \div 250 \approx 0.224$$

(ii) Determine the 95% confidence interval for the proportion of households who do the majority of their grocery shopping at Big Foods. Give your answer to four decimal places. (3 marks)

$$s = 0.0264, z_{0.95} = 1.96$$

$$(0.1723, 0.2757)$$

(iii) What is the margin of error of the 95% confidence interval? Give your answer to four decimal places. (1 mark)

$$E = 0.0517$$

An independent research company conducted a large-scale survey of household supermarket preferences and estimated that the true proportion of households that conduct most of their grocery shopping at Big Foods was 0.17 (assume that this is indeed the true proportion).

(b) With reference to your answer to part (a)(ii), does this result suggest that the junior staff member at Big Foods made a mistake? (2 marks)

No. The interval in (a)(ii) doesn't contain the true proportion, but that is to be expected in 5% of such samples. The CI is valid because the sample was random, it was sufficiently large to assume normality and we can assume independence of households selected.

VIC METH 2018 P2 Q4

Question 4 (16 marks)

Doctors are studying the resting heart rate of adults in two neighbouring towns: Mathsland and Statsville. Resting heart rate is measured in beats per minute (bpm).

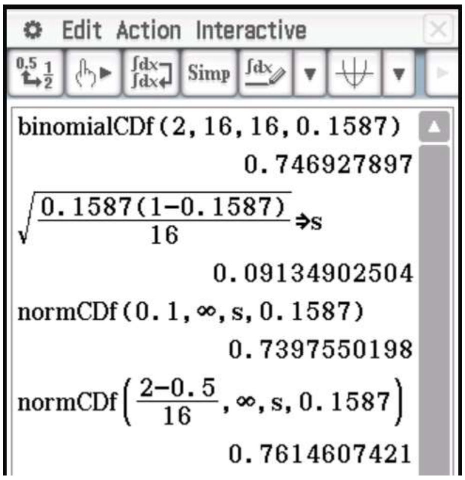
The resting heart rate of adults in Mathsland is known to be normally distributed with a mean of 68 bpm and a standard deviation of 8 bpm.

The doctors consider a person to have a slow heart rate if the person's resting heart rate is less than 60 bpm. The probability that a randomly chosen Mathsland adult has a slow heart rate is 0.1587

c. **i.** Find the probability that a random sample of 16 Mathsland adults will contain exactly one person with a slow heart rate. Give your answer correct to three decimal places. 2 marks

ii. For random samples of 16 Mathsland adults, $\hat{P}$ is the random variable that represents the proportion of people who have a slow heart rate.

Find the probability that $\hat{P}$ is greater than 10%, correct to three decimal places. 2 marks



Question 4ci.

Marks	0	1	2	Average
%	25	9	66	1.4

$X \sim \text{Bi}(16, 0.1587)$, $\Pr(X = 1) = 0.190$, correct to three decimal places

This question was reasonably well done. A method was required to get full marks. Stating the correct *n* and *p* value was sufficient. Some students gave their answer as 0.19.

Question 4cii.

Marks	0	1	2	Average
%	54	9	36	0.8

$\Pr(\hat{P} > 0.1) = \Pr(X > 1.6) = \Pr(X \geq 2) = 0.747$, correct to three decimal places

Some students used the normal approximation to the binomial distribution. There was poor use of variables, for example, $\Pr(\hat{P} > 0.1) = \Pr(\hat{P} > 1.6) = \Pr(\hat{P} \geq 2)$.

The doctors took a large random sample of adults from the population of Statsville and calculated an approximate 95% confidence interval for the proportion of Statsville adults who have a slow heart rate. The confidence interval they obtained was (0.102, 0.145).

d. **i.** Determine the sample proportion used in the calculation of this confidence interval. 1 mark

ii. Explain why this confidence interval suggests that the proportion of adults with a slow heart rate in Statsville could be different from the proportion in Mathsland. 1 mark

Question 17 **(14 marks)**

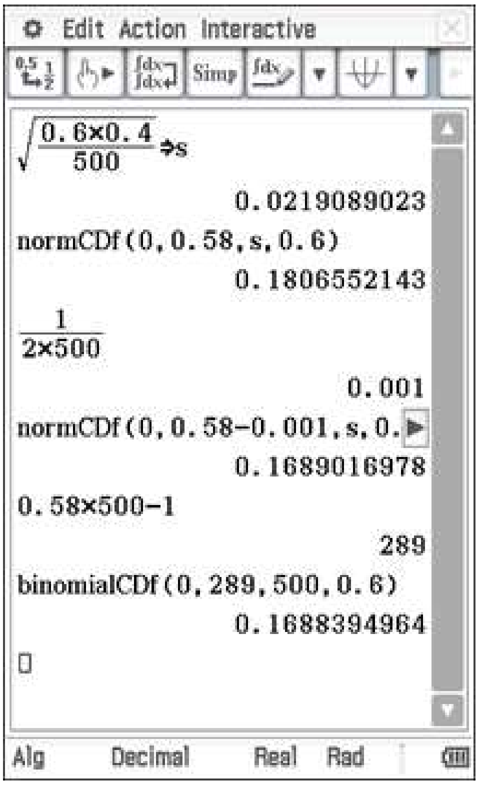
Tina believes that approximately 60% of the mangoes she produces on her farm are large. She takes a random sample of 500 mangoes from a day's picking.

(a) Assuming Tina is correct and 60% of the mangoes her farm produces are large, what is the approximate probability distribution of the sample proportion of large mangoes in her sample? (3 marks)

Solution
$\hat{p} \sim N\left(0.6, \frac{0.6 \times 0.4}{500}\right)$ That is, $\hat{p} \sim N(0.6, 0.02191^2)$
Specific behaviours
✓ states the distribution as normal ✓ gives the correct value of the mean ✓ gives the correct value of the variance (or standard deviation)

(b) What is the probability that the sample proportion of large mangoes is less than 0.58? (2 marks)

Solution
$P(\hat{p} < 0.58) = P\left(Z < \frac{0.58 - 0.6}{\sqrt{0.6 \times 0.4/500}}\right) = P(Z < -0.9129) = 0.18066$
Specific behaviours
✓ calculates the z-value correctly ✓ obtains the correct probability



$X \sim B(500, 0.6)$ and $P(\hat{p} < 0.58) = P(X < 0.58 \times 500) = P(X \leq 289) = 0.1688$

59% Question 17 attempted by 4343 candidates Mean 8.32(/14) Max 14 Min 0
Part (a) was done poorly generally. Candidates did not recognise that the distribution was a normal distribution and if they did, struggled to identify the parameters of the distribution. Part (c) was answered poorly, with many comments not making sense. Part (g) was not done well with reference to the marking key of 'taking another sample and obtaining another 95% confidence interval'.

Spec U4 CI Mean

Confidence intervals for means:

- understand the concept of an interval estimate for a parameter associated with a random variable
- examine the approximate confidence interval $\left(\overline{\mathrm{X}}-\frac{\mathrm{zs}}{\sqrt{n}},\ \overline{\mathrm{X}}+\frac{\mathrm{zs}}{\sqrt{n}}\right)$, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ
- use $\overline{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ
- collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality.

WA SPEC ATAR EXAM 2018 - Q16

Question 16 58% 58% average score on this question compared to exam average of 63%. **(9 marks)**

Tom wants to estimate the population mean number of hours, μ, worked by Australians per week. He takes a random sample of 400 workers and determines a 99% confidence interval for μ. The upper limit of this interval is 40.62 hours and the width of this interval is 1.08 hours.

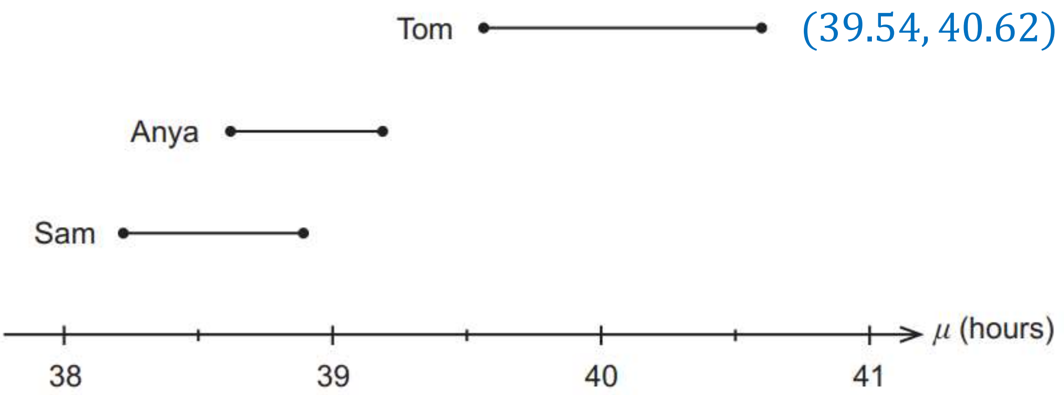
(a) Determine the sample mean for this sample of 400 workers. (2 marks)

$$\bar{X} = 40.62 - (1.08 \div 2) = 40.08$$

(b) Calculate, correct to 0.01 hours, the sample standard deviation for the sample of 400 workers. (3 marks)

$$0.54 = 2.5758 \times \frac{s}{\sqrt{400}} \Rightarrow s = 4.19$$

Two of Tom's colleagues, Anya and Sam, each take different random samples of size 400 and similarly determine 99% confidence intervals for the population mean μ. These confidence intervals, together with Tom's, are shown below.



(c) Anya makes the following statements based on these confidence intervals. Indicate **why** each of her statements is true or false.

(i) 'Tom's sample has a larger standard deviation compared with that of Sam's and mine.' (1 mark)

True - interval is wider

(ii) 'Tom's method of sampling must be biased since his confidence interval does not overlap with mine or Sam's.' (1 mark)

False - intervals are constructed from random samples and so may or may not overlap

(d) Which of these three confidence intervals contains the value for μ? Justify your answer. (2 marks)

No way to tell. We do not know μ and intervals are constructed from random samples.