

Transformation matrices

Transformation	Matrix
Reflection in the x-axis.	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Reflection in the y-axis	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection in the line $y = x$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Reflection in the line $y = -x$	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
<i>NB for reflections, mirror line is invariant line.</i>	
One-way stretch parallel to x-axis, factor p. (Invariant line is x-axis)	$\begin{bmatrix} p & 0 \\ 0 & 1 \end{bmatrix}$
One-way stretch parallel to y-axis, factor q. (Invariant line is y-axis)	$\begin{bmatrix} 1 & 0 \\ 0 & q \end{bmatrix}$
Scaling (two-way stretch) by factor p in x-direction and factor q in y-direction.	$\begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix}$
Dilation about (0,0), scale factor f ($ f > 1$: Enlargement, $ f < 1$: Reduction) (f negative, dilation 'back' thru' origin)	$\begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}$
Rotation <i>anticlockwise</i> about the origin of angle 90°	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
Rotation about the origin of angle 180°	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
Rotation <i>anticlockwise</i> about the origin of angle 270°	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
Rotation <i>anticlockwise</i> about the origin of angle θ from the x-axis.	$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$
Shear parallel to x-axis, factor k (Invariant line is x-axis)	$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$
Shear parallel to y-axis, factor m (Invariant line is y-axis)	$\begin{bmatrix} 1 & 0 \\ m & 1 \end{bmatrix}$

- Premultiply the object coordinate matrix by the transformation matrix to obtain the image coordinate matrix.
- Transformation AB means first transformation B followed by transformation A.
- A singular matrix (ie determinant = 0) will transform all points onto a straight line through the origin.