3

MATHEMATICS APPLICATIONS

Section One: Calculator-free

35% (55 Marks)

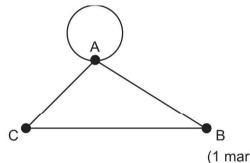
This section has **six** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 (8 marks)

The graph shown represents three buildings A, B and C, with connecting walkways, at a local school.



Why is the graph planar? (a)

(1 mark)

Planar graph

EAAS IT WILL BE CUT OFF

 \equiv

DO NOT WRITE

A planar graph is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.

Show that the graph satisfies Euler's formula. (b)

(2 marks)

Construct the adjacency matrix for the graph. (c)

(3 marks)

A student wishes to carry out closed walks of length two from Building A.

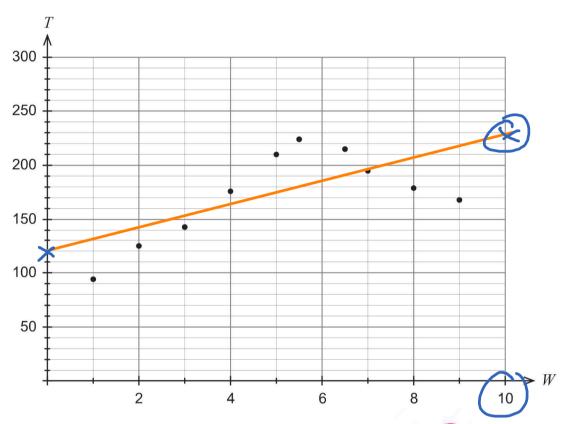
(d) List all his possible walks. (2 marks)

CALCULATOR-FREE

Question 2

(9 marks)

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water (W), in millimetres, used during each week and recorded the total number of tomatoes (*T*) produced by each plant. The scatterplot showing her results is drawn below.



Katie determined the following summary information:

- the equation of the least-squares line is T = 10.55W + 119.11

Identify the response variable. (a)

(1 mark)

Use the equation of the least-squares Ine to predict the total number of tomatoes (b) produced when 10 millimetres of water are given to a plant during each week. (2 marks)

10-15(10) 105-5



CALCULATOR-FREE 5 MATHEMATICS APPLICATIONS

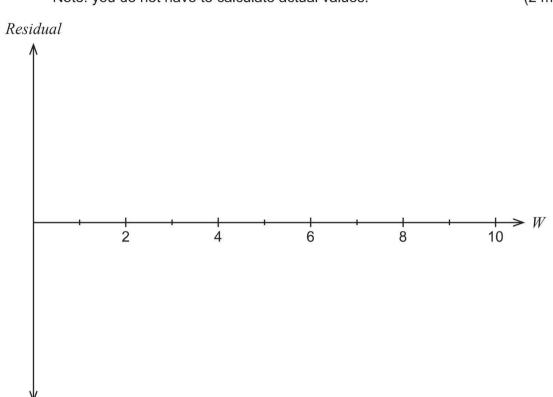
(c) Fit the least-squares line to the scatterplot.

(2 marks)

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data. Note: you do not have to calculate actual values.

(2 marks)



(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Massive number of marks for HA

MATHEMATICS APPLICATIONS

6

CALCULATOR-FREE

Question 3

(10 marks)

A company has four small workshops that each produce four different types of outdoor furniture. The annual cost of production of the furniture at each workshop is shown in the table below, with all values in thousands of dollars.

	Type 1 \$'000	Type 2 \$'000	Type 3 \$'000	Type 4 \$'000
Workshop A	25	43	50	39
Workshop B	33	31	56	39
Workshop C	28	47	59	38
Workshop D	36	32	56	41

The cost matrix is given by

25	43	50	39
33	31	56	39
28	47	59	38
_36	32	56	41



The company is interested in knowing what the minimum annual cost would be if each furniture type was allocated to its own individual workshop. The Hungarian Algorithm is to be used to determine the allocation and the minimum annual cost. The first step of the Hungarian Algorithm, where the smallest number in each row is subtracted from all other numbers in that row, is shown below.

(a) Continue the steps of the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type and state the **minimum** annual cost. (5 marks)

0	18	25	14
2	0	25	8
0	19	31	10
4	0	24	9 _

Type	Type 1	Type 2	Type 3	Type 4
Workshop				

Total minimum annual cost

Assignment problems

Thursday, 23 January 2020 4:17 PM

Assignment problems

- 4.3.10 use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem
- 4.3.11 determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems

Hungarian algorithm	The Hungarian algorithm is used to solve assignment (allocation) problems.

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CALCULATOR-FREE

MATHEMATICS APPLICATIONS

The revenue matrix, in thousands of dollars, for the sale of the furniture produced annually at each workshop is given by

7

(b) Given that Profit = Revenue - Cost, complete the Profit matrix below. (1 mark)

Profit matrix =
$$\begin{bmatrix} 12 & 18 & 10 & 14 \\ 12 & 21 & 17 & 11 \\ 10 & 18 & & & \\ \end{bmatrix}$$

(c) Use the Hungarian Algorithm to determine the appropriate allocation of workshops to furniture type that will produce the **maximum** annual profit. (4 marks)

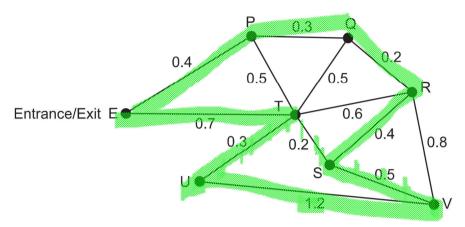
Туре	Type 1	Type 2	Type 3	Type 4
Workshop				

8

CALCULATOR-FREE

Question 4 (11 marks)

A marine park has attractions with paths connecting them. The vertices on the graph represent the attractions and the numbers on the edges represent the path distances (km) between the attractions. Visitors can either walk around the park or take one of the many shuttle buses that run between attractions.



The manager of the marine park leaves his office, which is located at the entrance/exit (E) and walks to attraction V.

(a) (i) Determine the shortest distance from E to V.

(1 mark)

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(ii) If the manager needs to pick up some tools left at U on the way, determine the route he should take and the corresponding distance, given he wants to take the shortest route from E to V. (2 marks)

Rachel arrives at the entrance. She wants to complete a Hamiltonian cycle.



(b) State the route she should take.

(2 marks)

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CALCULATOR-FREE

MATHEMATICS APPLICATIONS

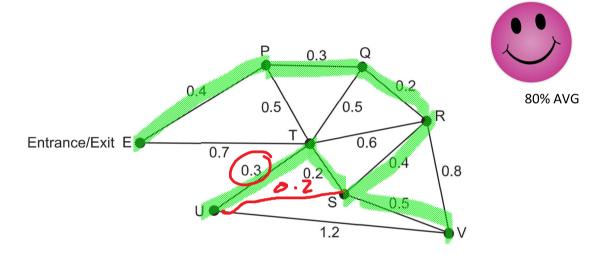
Drinking water is already being supplied at E. The manager has recently received funding to establish drinking fountains at each attraction. For this to happen, pipelines will need to be laid along the paths to each attraction. He has drawn up a table to show the distances between attractions.

9

	E	Р	Q	R	S	Т	U	V
Е	_	0.4	_	_	_	0.7	_	_
Р	0.4	-	0.3	_	-	0.5	I	_
Q	_	0.3	_	0.2	_	0.5	_	_
R	_	_	0.2	_	0.4	0.6	_	0.8
S	_	-	_	0.4	-	0.2	1	0.5
Т	0.7	0.5	0.5	0.6	0.2	_	0.3	-
U	_	-	-	-	-	0.3	-	1.2
V	_	_	_	0.8	0.5	_	1.2	_

Why create a table? Not needed for Prim's Alg.

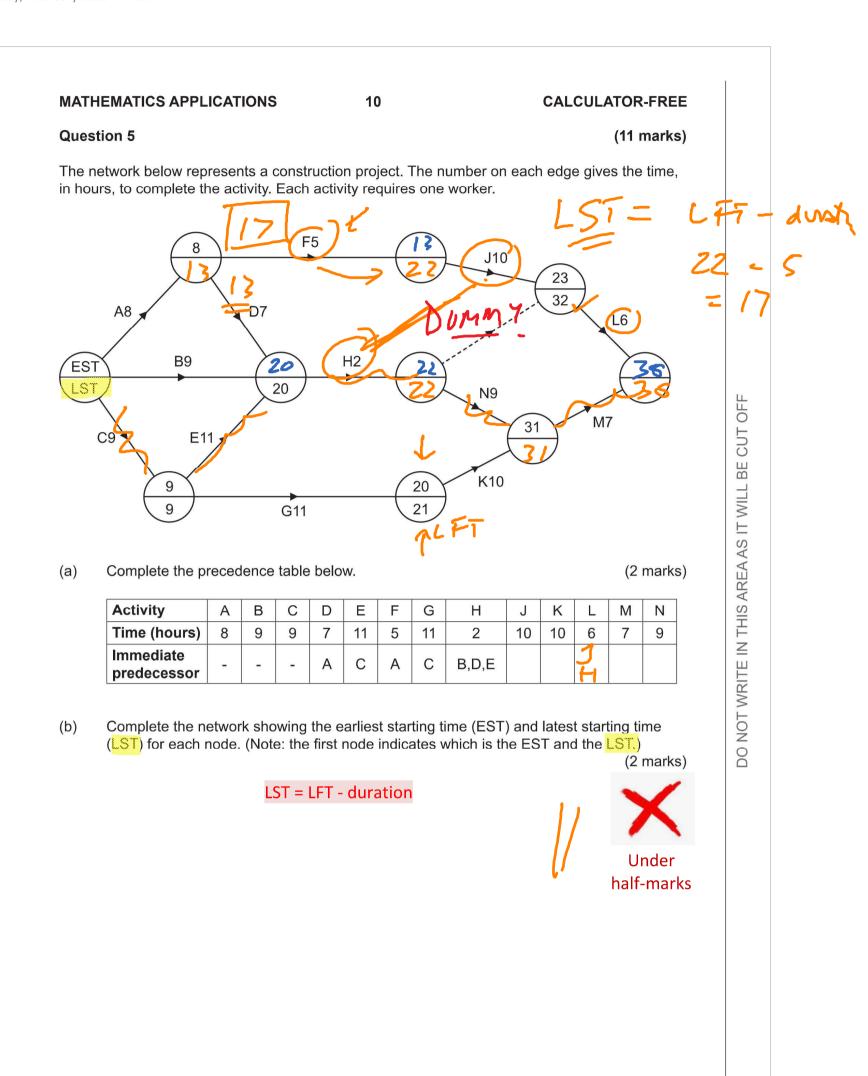
(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)



(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

Prim's algorithm

Prim's algorithm is an algorithm for determining a minimum spanning tree in a connected weighted graph.



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CALCULATOR-FREE 11 MATHEMATICS APPLICATIONS

- (c) Determine the critical path and the minimum completion time for the project. (2 marks)
- (d) Calculate the float times for Activities D and F. (2 marks)

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time.





(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)



(g) Explain the purpose of the dotted line on the network.

(1 mark)

1 500+ non attempts



12

CALCULATOR-FREE

Question 6

Examiner is a slow learner... here we go again modelling a discrete quantity with a continuous model...

(6 marks)

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule for the turtle population. (2 marks)

Tn+, = Tn x0-5 +20

(b) Determine the long-term steady state of the turtle population.

(2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

2 = 40

If the wildlife sanctuary preferred a long-term steady state of 80 Jurtles, what yearly (c) addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

1800+ non attempts

 $80 \times 2 \times 2 \times 1 \times 10^{-10}$

End of section

2

CALCULATOR-FREE

Section One: Calculator-free

35% (55 Marks)

Question 1

(8 marks)

(a) Why is the graph planar?

Can be drawn in the plane

(1 mark)

Solution

No two edges cross

Specific behaviours

√ states correct reason

(b) Show that the graph satisfies Euler's formula.

(2 marks)

Solution

v = 3, e = 4, f = 3. 3 + 3 - 4 = 2, verified.

Specific behaviours

 \checkmark gives correct values for the number of vertices, edges and faces

✓ correctly verifies Euler's formula

(c) Construct the adjacency matrix for the graph.

(3 marks)

Solution

 $\begin{array}{c|cccc}
A & B & C \\
A & 1 & 1 & 1 \\
B & 1 & 0 & 1
\end{array}$

 $C \mid 1 \mid 1 \mid 0$

Specific behaviours

- \checkmark gives correct entry for A to A
- √ gives the remainder of correct entries
- √ correctly labels rows and columns

A student wishes to carry out closed walks of length two from Building A.

(d) List all his possible walks.

(2 marks)

Solution

A-B-A A-C-A

A-C-A A-A-A

A total of 3 walks

Specific behaviours

- √ lists at least two walks
- √ lists all 3 walks

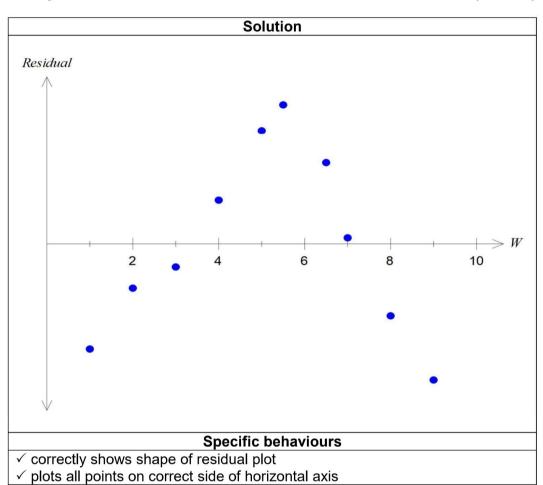
MATHEMATICS APPLICATIONS

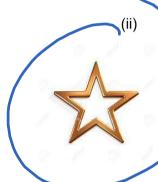
Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data.

Note: you do not have to calculate actual values. (2 marks)

5





Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Solution

A linear model is not appropriate as a pattern is evident in the residual plot.

Specific behaviours

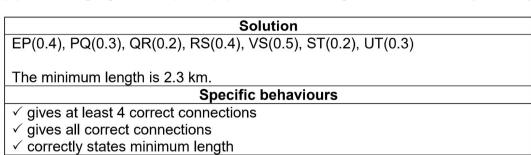
✓ states linear model is not appropriate

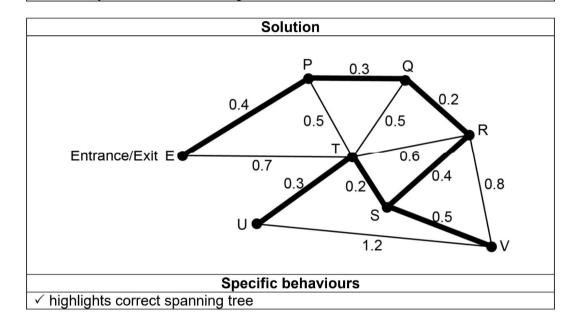
states a valid reason

MATHEMATICS APPLICATIONS

(c) (i) Use Prim's algorithm, or otherwise, to determine the minimum total length of pipelines. Highlight the required pipelines on the diagram below. (4 marks)

11





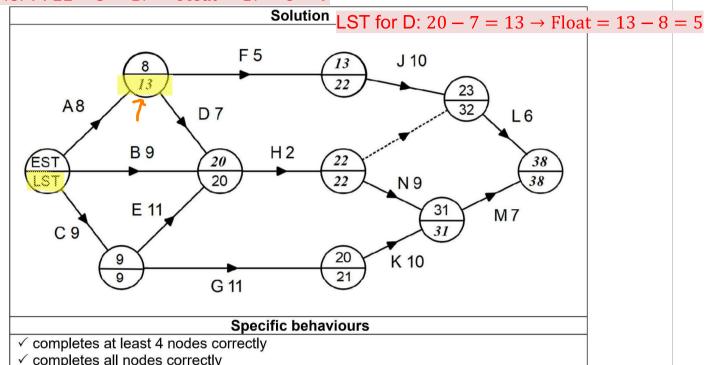
(ii) The manager has been told that a pipeline of length 0.2 km could be laid from S to U. How, if at all, will this affect the total length of pipelines that should be laid in order to maintain a minimum length? (2 marks)

Solution		
The minimum length will decrease by 0.1 km (as SU would be used instead of		
TU).		
Specific behaviours		
✓ states it is a decrease		
✓ gives the decrease as 0.1 km		

13 **MATHEMATICS APPLICATIONS**

Complete the network showing the earliest starting time (EST) and latest starting time (LST) for each node. (Note: the first node indicates which is the EST and the LST.)

LST for F: $22 - 5 = 17 \rightarrow \text{Float} = 17 - 8 = 9$



√ completes all nodes correctly

(c) Determine the critical path and the minimum completion time for the project. (2 marks)

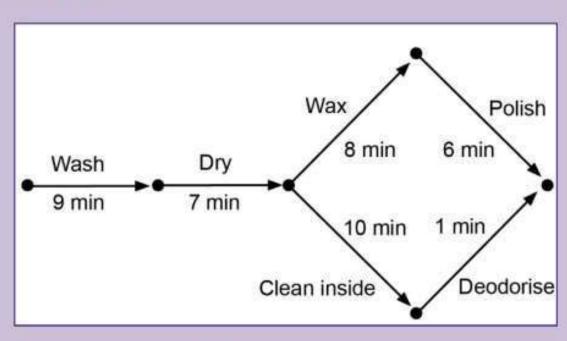
Solution		
Critical path is CEHNM. Minimum completion time is 38 hours		
Specific behaviours		
✓ states correct path		
✓ states correct time		

(d) Calculate the float times for Activities D and F. (2 marks)

	Solution		
Float time for D is 5 hours			
Float time for F is 9 hours			
	Specific behaviours		
✓ gives correct float for D			
✓ gives correct float for F			

(CPA) A project often involves many related activities, some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram.

> The network diagram below can be used to schedule the activities of two or more individuals involved in cleaning and polishing a car. The completion times for each activity are also shown.



Critical path analysis is a method for determining the longest path (the critical path) in such a network, and hence, the minimum time in which the project can be completed. There may be more than one critical path in the network. In this

project the critical path is 'Wash-Dry-Wax-Polish', with a total completion time of 30 minutes.

The earliest starting time (EST) of an activity, 'Polish', is 24 minutes because activities 'Wash', 'Dry' and 'Wax' must be completed first. The process of systematically determining earliest starting times is called forward scanning.

The shortest time that the project can be completed is 30 minutes. Thus, the latest starting time (LST) for the activity 'De-odorise' is 29 minutes. The process of systematically determining latest starting times is called backward scanning.

Most texts use backward scanning to determine the LFT for all activities that meet at a vertex and then define LST for individual activities to be LST = LFT - duration.

An alternative definition exists that states: The latest start time is (not the latest time to start the activity but) the latest time by which an activity *must be ready to start*.

A universal definition of float time is Float = LST - EST

Float or slack

The amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks is referred to as float or slack. For example, the activity 'De-odorise' is said to have a float of 3 minutes because its EST (26 minutes) is three minutes before its LST (29 minutes). As a result, this activity can be started at any time between 26 and 29 minutes after the project started. All activities on a critical path have zero floats.

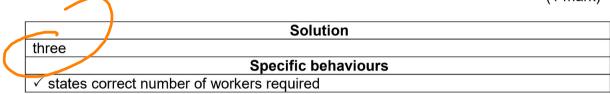
14

CALCULATOR-FREE

Question 5 (continued)

(e) Given that the sum of all the times of the activities is 104 hours, calculate the minimum number of workers required to complete the project in the minimum completion time.

(1 mark)



(f) What is the latest time into the project that Activity F could start without affecting the minimum completion time? (1 mark)

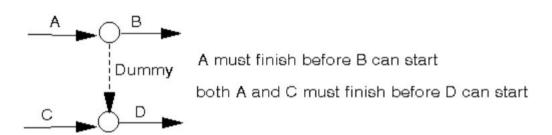
	Solution
Seventeen hours	
	Specific behaviours
✓ states correct latest time	

(g) Explain the purpose of the dotted line on the network.

(1 mark)

Solution		
Activity L depends on activities H and J		
Specific behaviours		
✓ states correct purpose		

It represents a dummy activity that is used because activities L and N share some immediate predecessors, but not all of them (N can start once H is complete but L can only start once H and J are complete).



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

CALCULATOR-ASSUMED

3

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

65% (100 Marks)

This section has 11 questions. Answer all questions. Write your answers in the spaces provided.

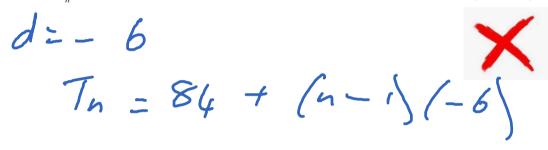
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 7 (6 marks)

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks)



(b) How many litres run out in the seventh minute?

(1 mark)

(c) How many litres have run out after eight minutes?

(1 mark)

Sum of terms... Avg: 0.33/1



(d) What is the capacity of the tank?

(2 marks)

Sum of terms... Avg: 1.09/2

AP Syllabus

Topic 3.2: Growth and decay in sequences (15 hours)

The arithmetic sequence

- 3.2.1 use recursion to generate an arithmetic sequence
- 3.2.2 display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations
- 3.2.3 deduce a rule for the n^{th} term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions
- 3.2.4 use arithmetic sequences to model and analyse practical situations involving linear growth or decay

Note use of 'sequence' and the absence of 'SERIES'.

1

CALCULATOR-ASSUMED

Question 8 (13 marks)

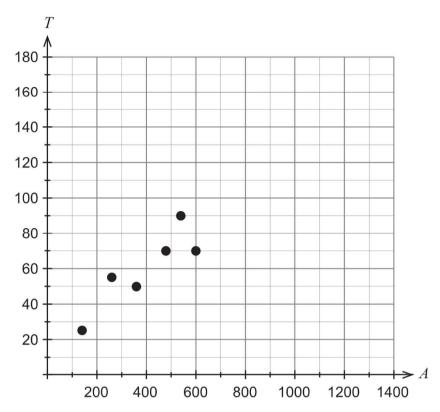
Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where A is the area of the lawn in square metres and T is the time in minutes. (Note: some values are missing.)

Customer	Α	В	С	D	E	F	G	Н
A (m^2)		260		480	540	600	860	1180
T (min)	25	55	50	70	90	70	135	140

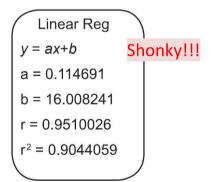
(a) Complete the scatterplot below.

(1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF



(b) From the information below, determine the equation of the least-squares line in terms of A and T and state the coefficient of determination for these data. (2 marks)



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(f)

CALCULATOR-ASSUMED

MATHEMATICS APPLICATIONS

(c) Interpret the value of the gradient of the least-squares line in the context of the question.

(2 marks)

5

(d) Given that Abdul charges \$30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m². (2 marks)

- (e) Explain whether the estimate determined in part (d) would be valid. (2 marks)
 - Using the least-squares line correct to three decimal places



(i) calculate the residuals for Customers B and D.

(2 marks)

B: 9.1

D: -1.2

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

6

CALCULATOR-ASSUMED

Question 9

(9 marks)

Corbie and Grant are investigating the possibility of purchasing their own apartment. They will need to open a bank account to save for a deposit. They make an initial contribution of \$7000 into the account and add a further \$800 at the end of each month. The account has an interest rate of 2.6% per annum, compounded monthly.

How much would be in the account at the end of two years? (a)

(3 marks)





1 n4 = Tn x (1 + 0.026)

Q9 Model: Deposit made; Regular sum added.

Corbie and Grant are willing to pay \$280 000 for the apartment and will require an additional \$22 000 in fees associated with the purchase. A deposit of 20% of the total cost will be needed in order to qualify for a bank loan.

(b) Show that the required deposit is \$60 400. (i)

(1 mark)

How long would it take to save enough for the deposit? (ii)

(2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(c) If, at the end of two years, their parents agree to give them a total of \$10 000 as a reward for their dedicated saving effort, determine the minimum monthly contribution Corbie and Grant will need to make if they are to have enough for the full deposit after four years.

2 100+ non attempts

Loose wording...



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CALCULATOR-ASSUMED

7

MATHEMATICS APPLICATIONS

Question 10 (7 marks)

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $T_n = 1.34T_{n-1}$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34.

(1 mark)

(b) Write the rule for the n^{th} term of this sequence.

(1 mark)

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)



Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, \$114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

1 600+ non attempts

8

CALCULATOR-ASSUMED

Question 11 (13 marks)

Data for the total occupancy of rooms for each season of the year at a Perth hotel is shown below.

n	Year	Season	Total rooms occupied	Seasonal mean	4-point centred moving average	Total rooms occupied as a percent of seasonal mean
1		Spring	1770			106.59
2	2015/16	Summer	1904	1660.5		В
3		Autumn	1591		1644.375	95.81
4		Winter	1377		1622.5	82.93
5		Spring	1641		1618	101.91
6	2016/17	Summer	1858	1610.25	1614.75	115.39
7		Autumn	1601		1602.25	99.43
8		Winter	1341		1584.75	83.28
9		Spring	1577		1558	103.48
10	2017/18	Summer	Α	1524.0	1532.375	116.93
11		Autumn	1463		1526.875	96.00
12		Winter	1274		1525.125	83.60
13		Spring	1600		С	105.28
14	2018/19	Summer	1745	1519.75	1525.25	114.82
15		Autumn	1504			98.96
16		Winter	1230			80.93

(a) Calculate the value of **A**, **B** and **C**.

(3 marks)

CALCULATOR-ASSUMED

MATHEMATICS APPLICATIONS

(b) Complete the table showing the seasonal index for each season.

(1 mark)

Order!!!

 Summer
 Autumn
 Winter
 Spring

 1.1545
 0.8268
 1.0432

9

(c) Calculate the deseasonalised value for Winter 2017/18.

(2 marks)

(d) Comment on the effect the seasonal index had on the value found in part (c). (1 mark)

Nonsense...

(e) The least-squares line using deseasonalised data is R = -12.071n + 1681.25. Use this line to predict the total number of rooms occupied during Spring 2020/21. (2 marks



When a prediction was made for Spring 2020/21, using the least-squares line based on the 4-point centred moving averages, the answer was 1481.

(f) Explain why this is different from the answer obtained in part (e).

(1 mark)

1 800+ non attempts



The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

(g) Comment on the statement made by the hotel manager.

(2 marks)

1 600+ non attempts

Cause and effect... who knew!



(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)

2 200+ non attempts



FACEBOOK comments



Bradford Bederson MACE Discussion Space

7 November 2019 at 12:40 · 🚱

I thought I was doing ATAR Applications not ATAR Hotel Management



🖒 🕌 Amalia Gray, Georgia Jeffreys and 265 others

13 comments



Hayley Roche Nathaniel Ritchie changing my tisc preferences to hotel management at ecu



Mikhaila Hernan > WACE Discussion Space

7 November 2019 at 12:18 · (3)

Can someone please explain why scsa asked us to give tips to a fucking hotel manager who's business is declining ???



🚹 酱 Josiah Wong, Eamonn Lane and 64 others

16 comments



Joyce Low Shuhadah Kamaruzaman bitch i said other factor such as deteriorating quality of the hotel Imfao they bouta have fun with my paper



Kane McIntosh > WACE Discussion Space

7 November 2019 at 13:31 · (a)

@ hotel manager, your business is declining because who tf wants to holiday in PERTH 😵



🚹 酱 Mackenzie Holman, Jade Fraser and 81 others

3 comments



Fox Thompson > WACE Discussion Space

7 November 2019 at 12:46 · (3)

lowkey think some hotel owner that also works at scsa is looking for tips to revive his business



🚻 🝏 Jarrod Tonkin-Sloan, Kai Schlegl and 73 others



Shaylee Duff WACE Discussion Space

7 November 2019 at 12:14 · (a)

who else whipped out their economic knowledge when the hotel owner believed business was down due to the economy?





(1) Sarah Smith, Keilah Truscott and 6 others

1 comment

10

CALCULATOR-ASSUMED

Unfortunate use of the word 'path' - that has a precise meaning in graph theory!!

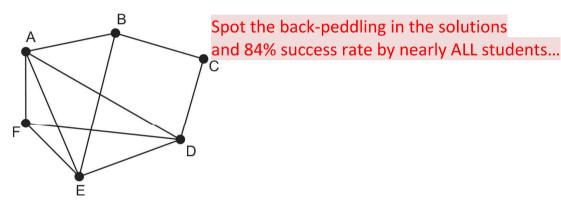
Question 12

Jake could have talked about footpaths... or tracks...

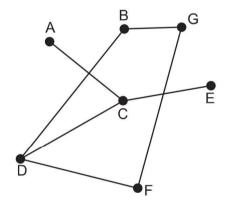
(6 marks)

Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.

- (a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.
 - (i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)



(ii) This park plan has been drawn as a bipartite graph. (3 marks)



Jake also makes the following incorrect statement in his presentation. 'A park plan can be a complete graph with 21 paths and six shelter huts'.

(b) If the plan must be a complete graph with 21 paths, how many shelter huts should Jake have quoted? (1 mark)

See next page

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CALCULATOR-ASSUMED

11

MATHEMATICS APPLICATIONS

Question 13

Deja vu?

(10 marks)

Mehmet has saved \$3600 from wages received at a part-time job. He is keen to invest this money in an account which earns 3.65% per annum, compounded monthly.

Over the next three years, Mehmet plans to continue working part-time and is aiming to make deposits of \$250 at the end of each month.

Q9 Model: Deposit made; Regular sum added.

(a) Write a recursive relation to give the value of the investment at the end of each month.

(2 marks)

Q13 Model: Deposit made; Regular sum added.

- (b) Mehmet hopes that this investment will double his initial savings in one year. Justify whether this is possible. (2 marks)
- (c) Determine the total amount of interest Mehmet would receive after three years.

(3 marks)



Unfortunately, after two years, Mehmet's working hours are reduced and he is only able to deposit \$120 at the end of each month.

(d) By how much would this reduce the value of his investment by the end of the three years? (3 marks)



12

CALCULATOR-ASSUMED

Question 14 (10 marks)

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

Internet access

		2014–15			2016–17	
State/territory	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
New South Wales	2407.9	414.5	2822.4	2439.9	421.8	2861.7
Victoria	1934.2 _	+ 305.1	Α	2008.2	305.8	2314.0
Queensland	1552.4	248.5	1800.9	1591.9	249.8	1841.7
South Australia	565.1	121.4	686.5	575.5 -	5) B	696.6
Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

(a) (i) Determine the value of **A** and **B** in the table above.

(2 marks)



(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

See next page

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CALCULATOR-ASSUMED

13

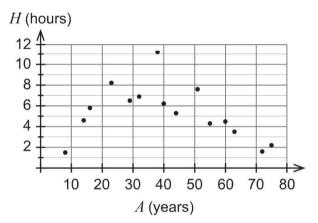
MATHEMATICS APPLICATIONS

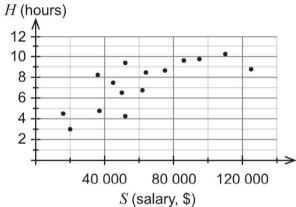
(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

An internet service provider from Tasmania wanted to determine whether a person's age, A, or salary, S, affected the number of hours, H, of internet usage per day. The graphs below each show the recorded data for people surveyed.

Daily Internet Usage According to Age

Daily Internet Usage According to Salary





- (b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)
 - (ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

-1₂₅, -0.95, -0.75, -0.3, 0.1, 0.3, 0.75, 0.95, 1.25

Choose the best estimate from the list for each of the graphs shown above.

(2 marks)

Graph	Correlation coefficient
Daily internet usage according to age	
Daily internet usage according to salary	

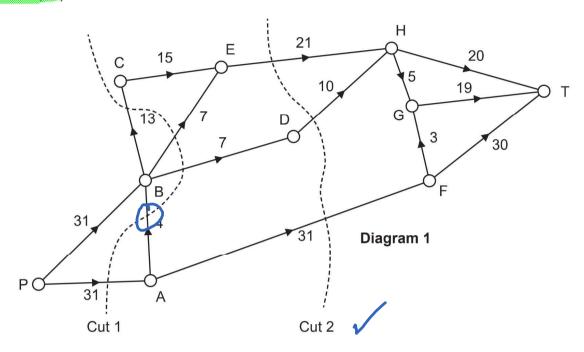
14

CALCULATOR-ASSUMED

Question 15

 $P = I_{XV}$ (11 marks)

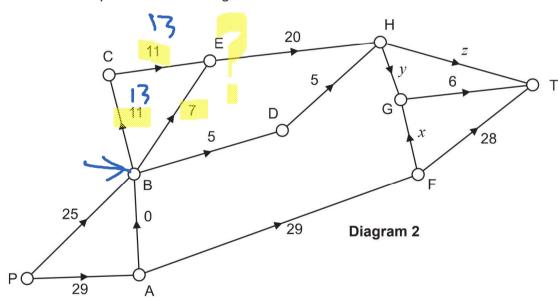
The directed network below shows the maximum available capacity for transferring power between different sub-stations on a small island. The number on each edge gives the capacity in kilovolts (kV).



(a) State the capacity of each cut in Diagram 1.

(2 marks)

Diagram 2 shows a possible flow through the same network.



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

CALCULATOR-ASSUMED 15 MATHEMATICS APPLICATIONS

(b) Determine the in Diagram 2.

(1 mark)

1 900+ non attempts

(c) Calculate the value of x, y and z in Diagram 2.

(3 marks)

2 200+ non attempts

(d) Determine the maximum flow for the original network (Diagram 1).

(2 marks)

Engineers wish to increase the maximum capacity to sub-station T. They propose to add a new transmission line from E to T of capacity 3 kV **or** a new transmission line from D to G of capacity 3 kV.

(e) Determine which of these proposals will increase the maximum capacity to sub-station T. Justify your answer. (3 marks)

2 600+ non attempts



16

CALCULATOR-ASSUMED

Question 16

(7 marks)

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (<i>T</i>)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25

7

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was T=-0.022A+24.97 and $r_{AT}=-0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value.

(2 marks)



(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

2 000+ non attempts

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

CALCULATOR-ASSUMED 17 MATHEMATICS APPLICATIONS

Question 17 Deja vu? (8 marks)

toel has set up a special investment fund that has a current balance of \$350,000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further \$355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is \$101,000 and he has just turned 60 years of age.

(a) Calculate Joel's total monthly contribution to the fund.

(2 marks)

6

(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)

2 100+ non attempts



Q17 Model: Deposit made; Regular sum added.

Q9 Model:

Q13 Model: Deposit made;

Deposit made;

Regular sum added.

Regular sum added.

Where did reducing balance loan go?

Joel retires at 67 and wants to use his lump sum payment to set up a regular income. He decides to look at two options that offer monthly payments.

Option 1: A reducing balance annuity at 7% per annum, compounded monthly.

Option 2: A perpetuity at 7.5% per annum, compounded monthly.

(c) Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks

3 400+ non attempts



(d) Calculate his monthly income using Option 2.

(2 marks)

3 700+ non attempts



End of questions

2

CALCULATOR-ASSUMED

Section Two: Calculator-assumed

65% (100 Marks)

Question 7 (6 marks)

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks)

	Solution
$T_n = 84 + (n-1)(-6)$	
=90-6n	
	Specific behaviours
\checkmark states correct value for A	
✓ states correct value for <i>B</i>	

(b) How many litres run out in the seventh minute?

(1 mark)

	Solution
48 L	
	Specific behaviours
√ states correct value	

(c) How many litres have run out after eight minutes?

(1 mark)

Solution
Sum of first eight terms is 504 L
Specific behaviours
✓ states correct value

Method to obtain sum of terms not indicated...

(d) What is the capacity of the tank?

(2 marks)

Solution
$T_{ m 15}=0$, Sum of first 15 terms is 630 L
Specific behaviours
✓ states that the 15 th term is zero
✓ states correct capacity

CALCULATOR-ASSUMED

MATHEMATICS APPLICATIONS

Explain whether the estimate determined in part (d) would be valid. (e) (2 marks)

5

Solution
Estimate would be valid since it is interpolation and the correlation coefficient is strong
Specific behaviours
✓ correctly explains validity

- ✓ explains validity with reference to either interpolation or correlation coefficient
- (f) Using the least-squares line correct to three decimal places
 - (i) calculate the residuals for Customers B and D.

(2 marks)

Solution
Residual for customer B is 9.092
Residual for customer D is –1.208
Specific behaviours
✓ correctly calculates residual for customer B
✓ correctly calculates residual for customer D

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

Solution
The change in sign indicates the residuals are above and below the least-
squares line
The size indicates that the residual for D is closer to the line than the residual for
customer B (or vice versa)
Specific behaviours
✓ states correct meaning of residual sign

✓ states correct meaning of residual size

This solution 'compares' the residuals but the question doesn't ask for this...

CALCULATOR-ASSUMED

MATHEMATICS APPLICATIONS

The manager of the hotel attended a meeting with the owners of the hotel. She explained to the owners that the reduction in occupancy was due to the downturn in the Western Australian economy in recent years.

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(g) Comment on the statement made by the hotel manager. (2 marks)

Solution

Cause not established

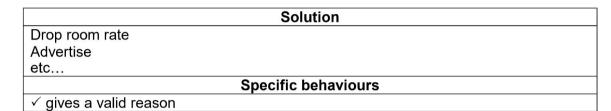
comment not appropriate

Specific behaviours

✓ states the cause in not established

✓ states the comment is not appropriate

(h) What practical advice, in the context of the question, would you give to the manager of the hotel? (1 mark)



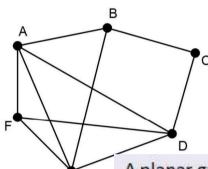
12

CALCULATOR-ASSUMED

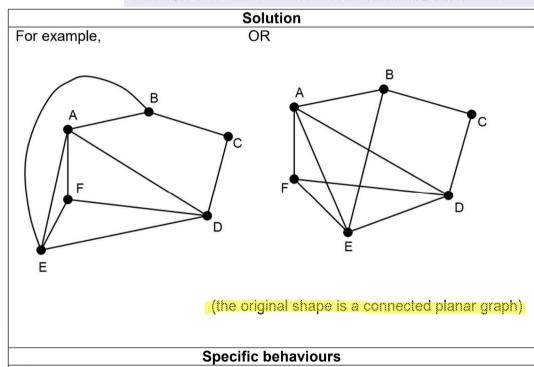
Question 12 (6 marks)

Jake, a park ranger, is giving a presentation at a National Park and Wildlife Conference on possible designs for a new park. Unfortunately, Jake made mathematical errors in his presentation about the paths (represented by edges) and shelter huts (represented by vertices) in the park.

- (a) For each of the following statements, the graph drawn by Jake was incorrect. Redraw the graph to match the statement correctly.
 - (i) This park plan has been drawn as a connected planar graph containing six vertices. (2 marks)



A planar graph is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.



- √ the graph is drawn with no two edges crossing
- ✓ the graph is drawn with all connections

OR

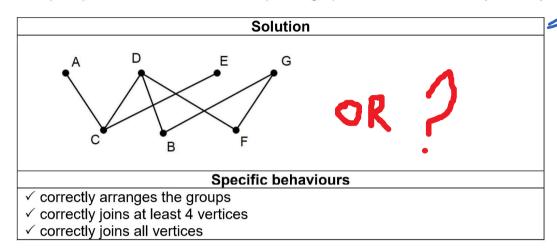
✓ correctly draws or states that the original is a connected planar graph

Tuesday, 7 January 2020 11:43 AM

A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.

> (ii) This park plan has been drawn as a bipartite graph.

(3 marks)



Jake also makes the following incorrect statement in his presentation. 'A park plan can be a complete graph with 21 paths and six shelter huts'.

If the plan must be a complete graph with 21 paths, how many shelter huts should Jake (b) have quoted? (1 mark)

Solution				
Correct number of huts is seven				
Specific behaviours				
✓ states correct number of huts				

16

CALCULATOR-ASSUMED

Question 14 (10 marks)

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

	2014–15			2016–17		
State/territory	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
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Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

Internet access

(a) (i) Determine the value of **A** and **B** in the table above.

(2 marks)

Solution		
A = 1934.2 + 305.1 = 2239.3		
B = 696.6 - 575.5 = 121.1		
Specific behaviours		
✓ correctly determines A		
✓ correctly determines B		

(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

Solution
$$2014-15: \frac{2407.9}{2822.4} = \frac{85.31\%}{2861.7}, 2016-17: \frac{2439.9}{2861.7} = \frac{85.26\%}{2861.7}$$

There is a slight decrease in the percentage of households with internet access from 2014–15 to 2016–17.

Specific behaviours

- √ calculates the correct percentages
- √ correctly rounds percentages to two decimal places
- ✓ states there is a small drop in the percentage of households with internet access from 2014–15 to 2016–17

CALCULATOR-ASSUMED

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MATHEMATICS APPLICATIONS

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice.

(3 marks)

Solution

 $T = -0.264 \times 22.93 + 30.94 = 24.89$

 $T = -0.022 \times 9 + 24.97 = 24.77$

The prediction using latitude is more valid as the correlation coefficient is much stronger.

Specific behaviours

- ✓ correctly determines a prediction using latitude and altitude
- √ correctly states that the prediction using latitude is more valid
- √ correctly explains that latitude has a stronger correlation coefficient

11:43 AM

MATHEMATICS APPLICATIONS

24

CALCULATOR-ASSUMED

Question 17 (8 marks)

Joel has set up a special investment fund that has a current balance of \$350 000. He contributes 7.5% of his monthly income to the investment and has an overseas pension which contributes a further \$355 per month. The investment fund has an interest rate of 6.5% per annum, compounded monthly. Joel's annual salary is \$101 000 and he has just turned 60 years of age.

(a) Calculate Joel's total monthly contribution to the fund. (2 marks)

Solution			
Monthly amount = $0.075 \times \frac{101000}{12} + 355 = \986.25			
Specific behaviours			
✓ calculates correct monthly income contribution			
✓ calculates correct total monthly contribution			

(b) Calculate the lump sum that he could receive if he retires on his 67th birthday. (2 marks)

Solution
$$N = 12 \times 7, I = 6.5, PV = 350\ 000, PMT = 986.25, P/Y = C/Y = 12$$

$$FV = \$655\ 539.45$$

$$Specific behaviours$$

$$\checkmark correctly uses positive (or negative) values for both PV and PMT$$

$$\checkmark calculates correct lump sum$$

$$T_{n+1} = T_n \left(1 + \frac{2.6}{1200} \right) + 800, T_0 = 7000$$

$$Wait a minute... Q13...$$

$$Valia minute... Q13...$$

$$Valia minute... Q13...$$

$$T_{n+1} = T_n + \frac{0.0365}{12} \times T_n + 250, T_0 = 3600$$

Joel retires at 67 and wants to use his lump sum payment to set up a regular incom $T_{n+1} = T_n \times 1.00541\overline{6} + 986.25$, $T_0 = 350\,000$

Option 1: A reducing balance annuity at 7% per annum, compounded monthly.

Option 2: A perpetuity at 7.5% per annum, compounded monthly.

Calculate his maximum monthly income for the next 20 years using Option 1. (2 marks) (c)



	Solution		
N = 12 × 20, I = 7, PV = 655 539.45, F	V = 0, $P/Y = C/Y = 12$		
PMT = \$5082.39			
Specific behaviours			
✓ uses correct values for N and FV			
✓ calculates correct monthly income			

(d) Calculate his monthly income using Option 2. (2 marks)

Solution		
N = (any positive integer), I = 7.5, PV = 655 539.45, FV = $-655 539.45$, P/Y = C/Y = 12		
PMT = \$4097.12		
Specific behaviours		
✓ uses correct values for PV and FV		
✓ calculates correct monthly income		